Risky Business: Estimating Value-at-Risk for Bitcoin

Sierra Laine *advised by Dr. Eliana Christou*

University of North Carolina at Charlotte

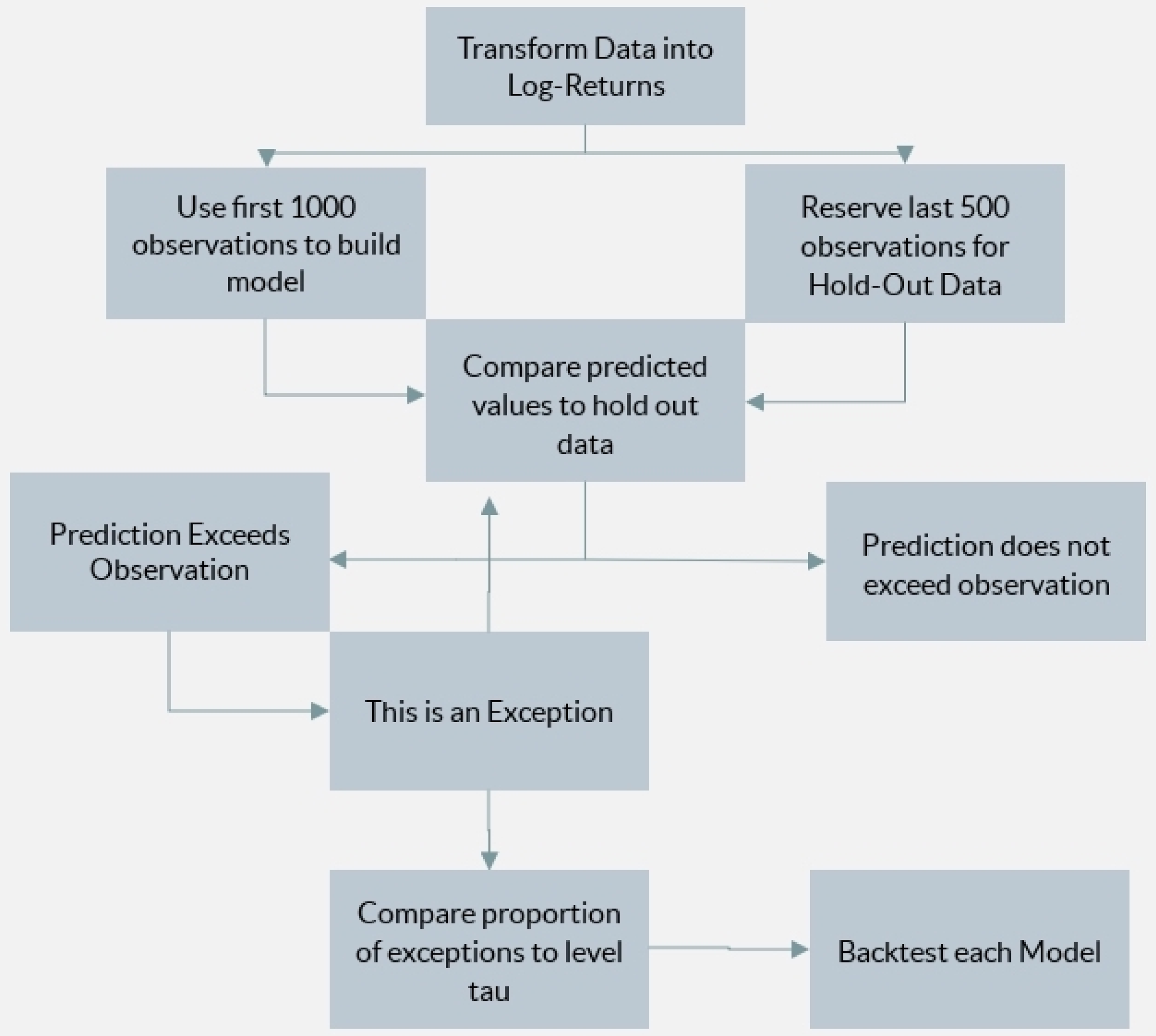
Introduction

Value-at-Risk (VaR) is one of the best known and most heavily used measures of financial risk.
Definition: VaR is the maximum possible loss of a given portfolio over a set time period and at a given significance level. In another words, VaR denotes the dollar amount one stands to lose for a given investment with a certain probability.

- **Who cares?**
  Since Basel II, financial institutions are compelled to calculate VaR to comply with banking regulation. However, VaR is also used by any company that is interested in Risk Management.

Project Design

This dataset consists of daily Bitcoin (BTC) historical prices from 2011-2016. The difference between consecutive prices are referred to as returns.



Existing Methods

The methods considered in this project are:

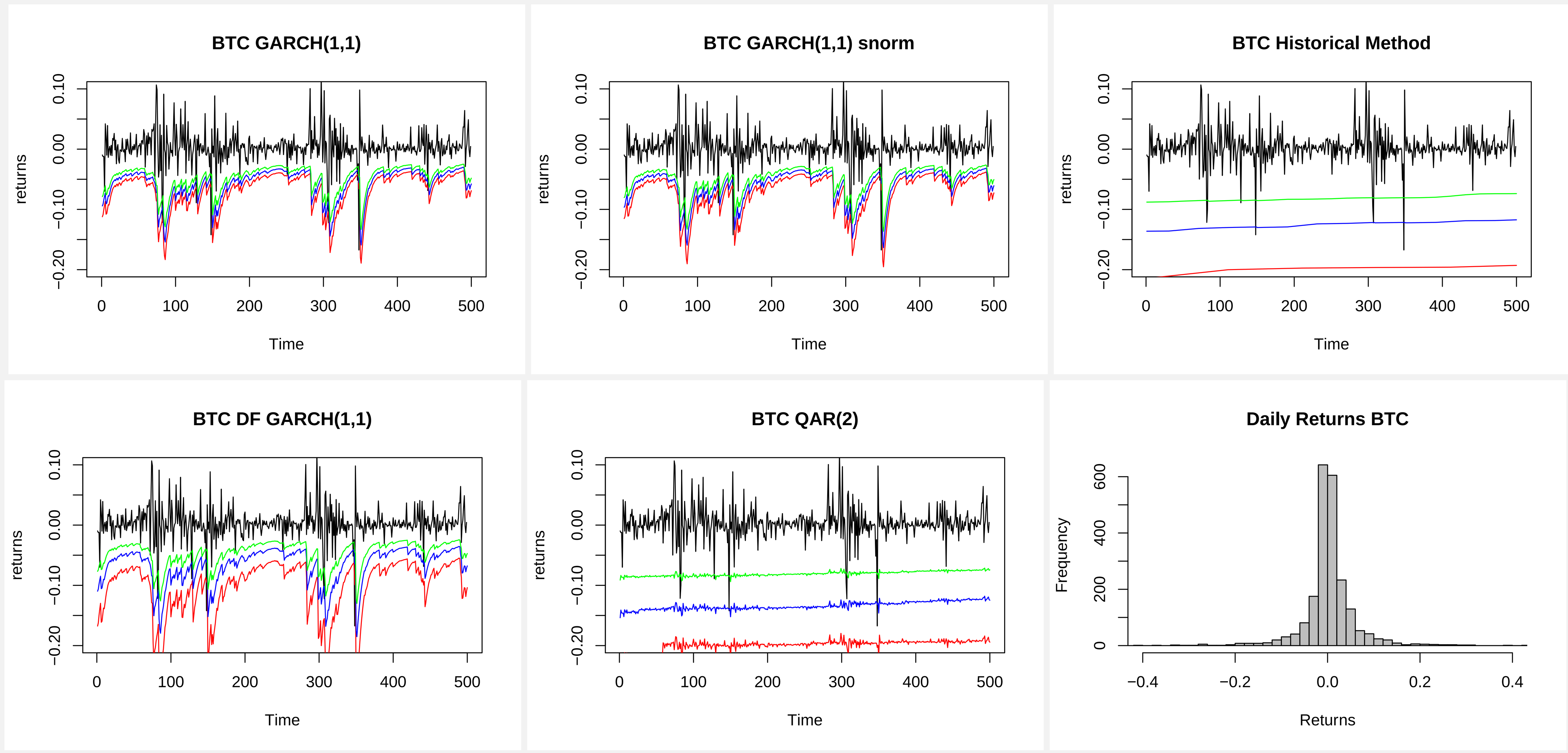
1. **GARCH(1,1)-$\mathcal{N}$**
2. **GARCH(1,1)-$\mathcal{SN}$**
3. **Historical Method**
4. **DFGARCH(1,1)**
5. **Quantile Autoregression (QAR)**

Results for BTC

Table: Predicted levels of τ

	GARCH(1,1)-$\mathcal{N}$	GARCH(1,1)-$\mathcal{SN}$	Historical	DFGARCH(1,1)	QAR(2)
τ =.01	0.012	0.012	0.000	0.004	0.000
τ =.025	0.018	0.018	0.006	0.012	0.004
τ =.05	0.020	0.020	0.014	0.020	0.014

highlighted values are closest to τ



Above are the time series of returns, along with the one-step ahead VaR forecast for different methods, for the 500 observations. We can see how the GARCH models capture the movement of the data, whereas the historical and QAR(2) methods appear more static. The last figure shows that the returns look slightly skewed and not perfectly normal.

The Math behind the Methods

For time series of returns $\{r_t\}_{t=1}^{n}$,

Historical Method: estimates VaR using empirical quantiles.

GARCH(1,1) Model: assumes $r_t = \sigma_t \epsilon_t$, $\sigma_t^2 = \omega + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

- ω, α_1, and β_1 are unknown parameters,
- $\epsilon_1, \epsilon_2, \ldots \overset{\text{i.i.d}}{\sim} \mathcal{N}$ or $\overset{\text{i.i.d}}{\sim} \mathcal{SN}$ or are distribution free.
- Then, $VaR_\tau(t) = -\sigma_t F^{-1}(\tau)$, where $F(\cdot)$ denotes the cumulative distribution function (cdf) of ϵ.

Quantile Regression Model: introduced by Koenker and Bassett (1978) and is used when interest lies on conditional quantiles.

- $Q_\tau(r_t|\mathscr{F}_{t-1}) = \beta_{0,\tau} + \sum_{i=1}^{p} \beta_{i,\tau} r_{t-i}$,
- $\beta_{i,\tau}$, $i = 0, 1, \ldots, p$, are unknown parameters. Here we use $p = 2$.
- Then, $VaR_\tau(t) = -Q_\tau(r_t|\mathscr{F}_{t-1})$.

Backtesting

What is backtesting?
Backtesting consists of three tests that determine the statistical significance of the model.

1. Unconditional Coverage (UC)
   - Measures whether or not the predicted value of τ, $\hat{\tau}$, is different from τ.
2. Independence (IND)
   - Independence is an assumption of our models. If independence between observations is not present, then our model is inherently flawed.
   - This test observes clustering of observations to determine independence.
3. Conditional Coverage (CC)
   - A combined test of the two above.
   - Indicates overall model health.

Table: Backtesting Results

	τ =.01	τ =.025	τ =.05
GARCH(1,1)-$\mathcal{N}$	—	—	UC,CC
GARCH(1,1)-$\mathcal{SN}$	—	—	UC,CC
Historical	UC	UC,CC	UC, IND, CC
DFGARCH(1,1)	—	UC	UC, CC
QAR(2)	UC, CC	UC, CC	UC, IND, CC

Key

- UC, IND, CC = Problem with Respective Test
- — = No Problem

Conclusion

- **Which is the best method?**
  For BTC, the GARCH(1,1) methods outperform the historical and quantile regression models. GARCH(1,1)-$\mathcal{N}$ and GARCH(1,1)-$\mathcal{SN}$ also perform better than DF GARCH(1,1).
  - However, other indices may be predicted better by other modelling methods
- **How can we be sure this model is good?**
  The GARCH(1,1) models perform well at τ =.01, .025 levels since there were no backtesting issues. Even at τ =.05 there are no issues with independence, but the forecasted VaR is not close to the level of τ. According to the backtesting results, all methods tend to forecast VaR poorly at τ=.05 for BTC from 2011-2016.

References

[1] Koenker, R. and Bassett, G. (1978) Regression quantiles. *Econometrica* 46, 33–50.