Comparing Model Performance Based on Value-at-Risk and Expected Shortfall

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Introduction

Financial Risk is defined as the chance of losing money on an investment. Value-at-Risk (VaR) and Expected Shortfall (ES) are quantities that help assess financial risk.
In this work we illustrate the utility of VaR and ES through comparing different mathematical models at different financial states, notably before and after COVID-19.

Key Terms

1. **Value-at-Risk (VaR)-** The maximum loss over a given time period and confidence level.
2. **Expected Shortfall (ES)-** The magnitude of losses that exceed the VaR.
3. **Exception-** When the actual loss is larger than the VaR.
4. **Proportion-** The number of exceptions experienced divided by the total number of returns.

What Datasets Were Used?

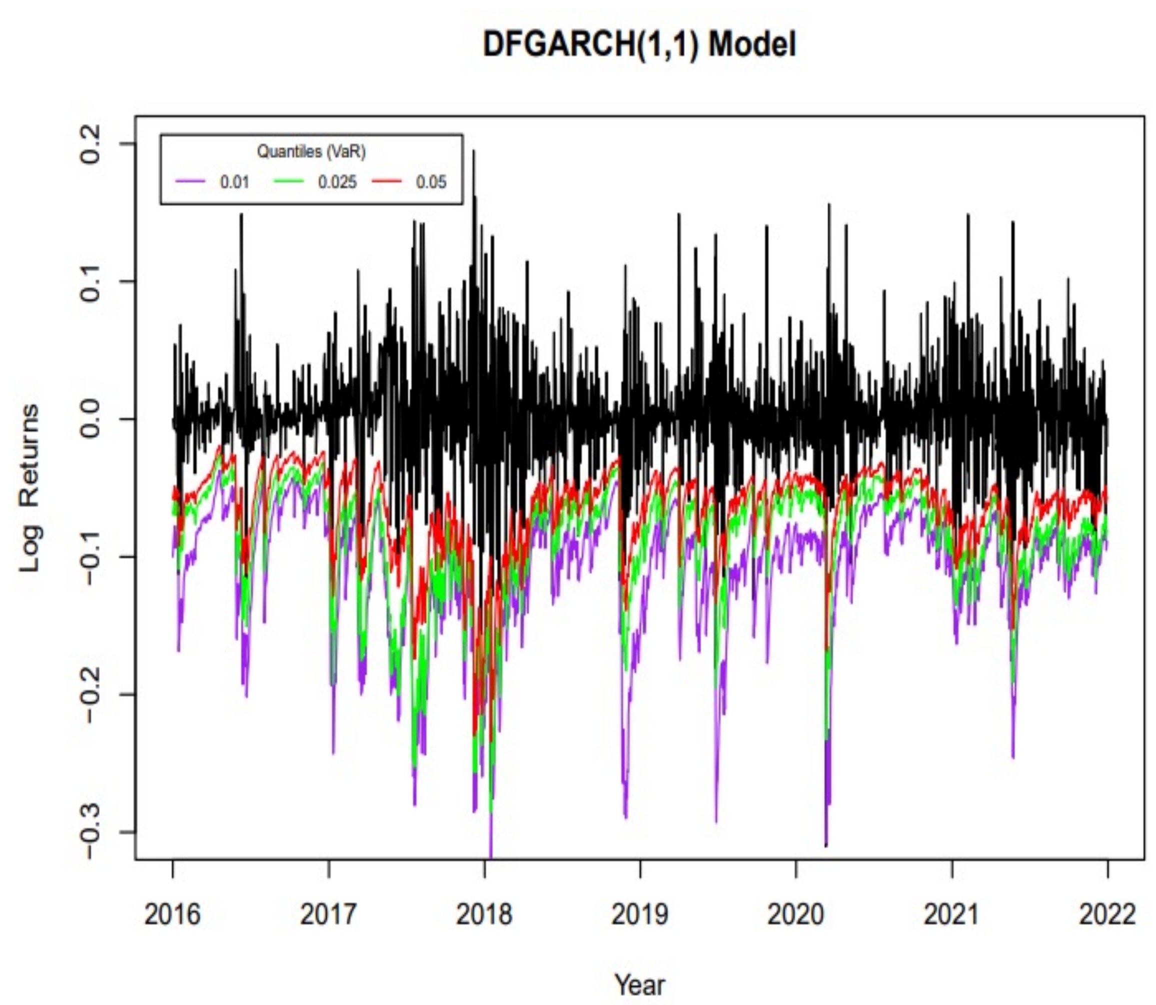
1. **S&P500** – Represents how well the stock market is performing based on returns of 500 companies in the U.S.
2. **Bitcoin** – A digital, decentralized currency that began in the early 2010's.
3. **U.S Dollar** – The official currency of the U.S.
4. **Chinese Yuan** – The unit of measure of the Chinese currency, renminbi.
5. **U.S. 30 Year Bond**- Securities that earn interest over 30 years.

What Models Were Compared?

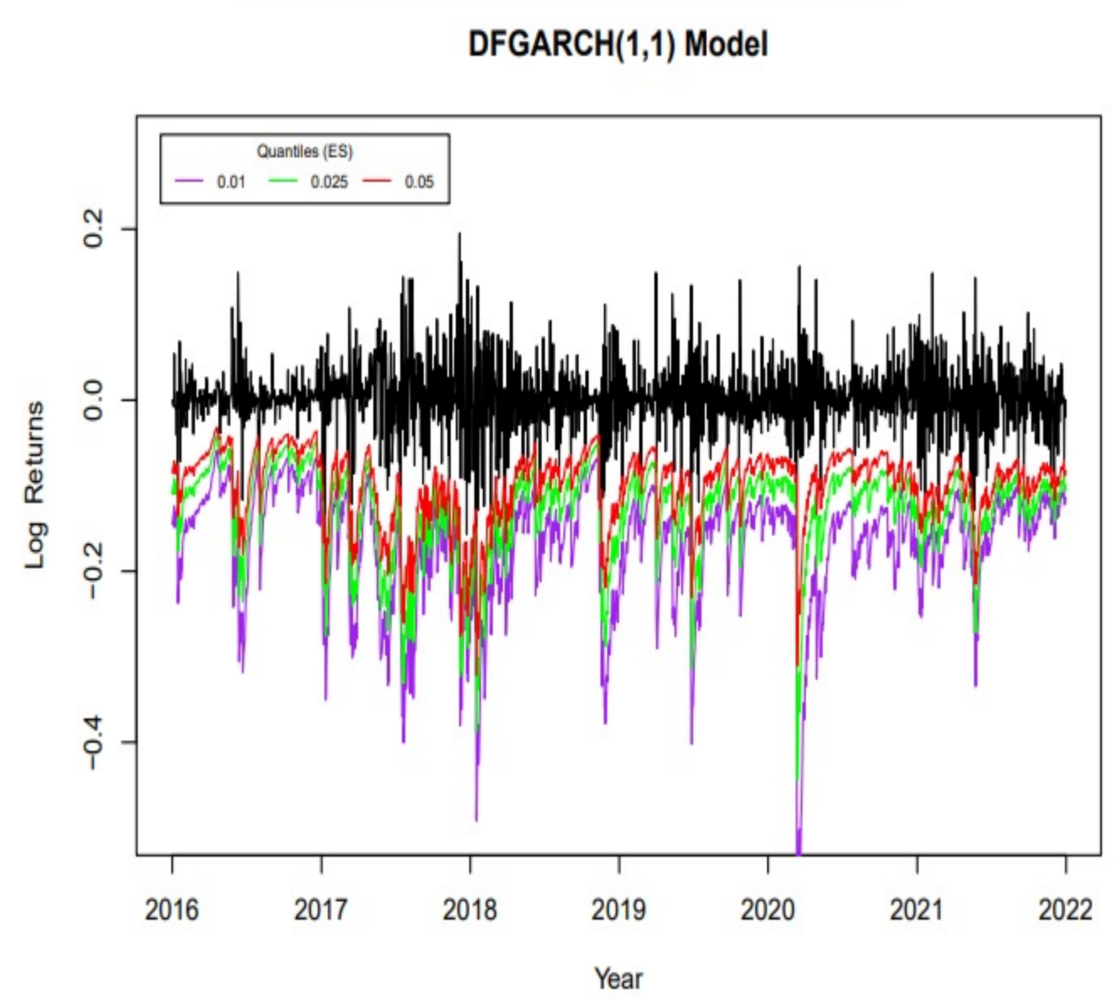
1. Historical Method
2. GARCH(1,1)
3. DFGARCH(1,1)
4. Quantile Autoregression (QAR(2))

Log Returns

Value-at-Risk



Expected Shortfall



Above are two time series graphs that illustrate the log returns of Bitcoin. The graph on the left is graphing the VaR at three different quantiles (0.01, 0.025, and 0.05). The graph on the right is for ES of the same log returns and quantiles.

Methodology

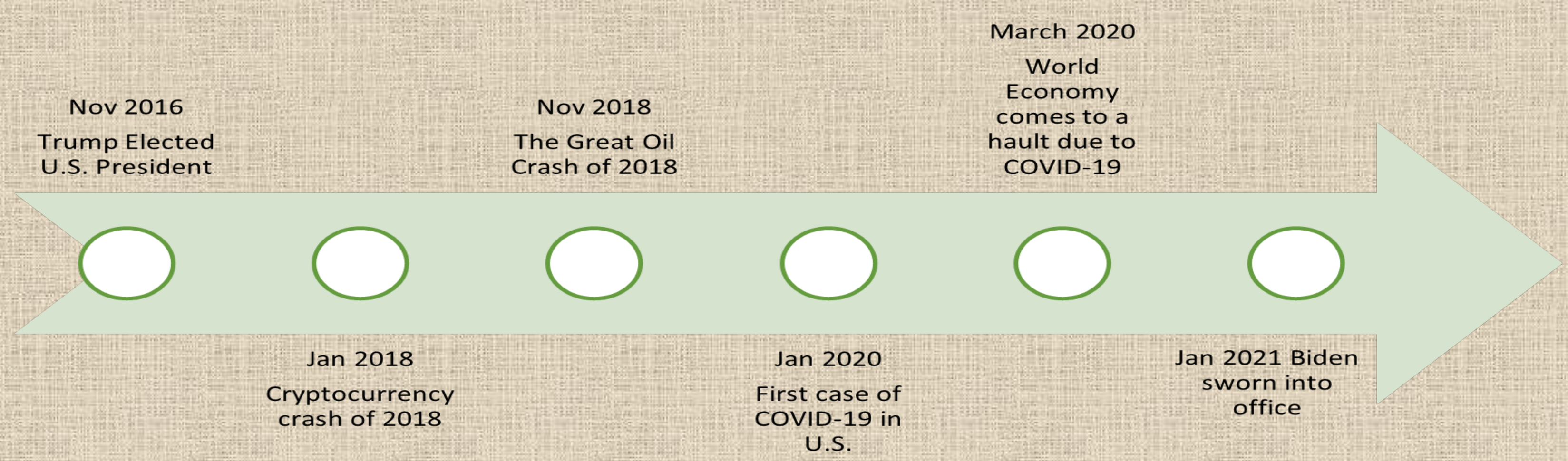
1. Historical Method- Calculates VaR and ES using empirical quantiles. This is done by ordering the returns from least to greatest and breaking them up by specific quantiles.

2. GARCH(1,1) - Assumes a normal distribution. VaR is just a quantile, so $\hat{\sigma}_t$ is approximated and multiplied by a z-score [2]. ES takes the exceptions, multiplies it by the negative return, and divides by quantile.

3. DFGARCH(1,1) - Does **not** assume a distribution like GARCH(1,1). $\hat{\sigma}_t$ is multiplied by empirical quantiles [2].

4. QAR(**2**) - Calculates VaR using the last **2** returns. The model uses data to approximate three coefficients at specific quantiles. These coefficients are used to find the VaR [4].

Notable Historical Events



Backtest Results

Table: Bitcoin Backtesting Result

Tau	GARCH(1,1)	DFGARCH(1,1)	Historical	QAR(2)
.01		✓		
uc	X			
ind				
cc	X			
Z		✓		
V		✓		
.025		✓		
uc	X			X
ind			X	
cc	X		X	X
Z		✓		
V		✓		
.05	✓			
uc				X
ind			X	
cc				X
Z		✓		
V				✓

Legend
✓ Best Proportion
X VaR Issue
✓ Best ES Value

Backtesting Terminology

- **Unconditional Coverage (uc)** Determines if the model proportion is "close" to tau.
- **Independence (ind)** Determines if exceptions are random occurrences.
- **Conditional Coverage (cc)** A combination of uc and ind.
- **Z** a method to test how close the estimator of ES is to the true ES value [1].
- **V** a method to test how close the estimator of ES is to the true ES value [3].

Conclusion

- **Proportions**: DFGARCH(1,1) was the most competitive, followed by QAR(2) or Historical.
- **VaR**: DFGARCH(1,1) was the most competitive because it lacked dependence issues in most of the datasets.
- **ES**: DFGARCH(1,1) and QAR(2) were the two most competitive models.

Therefore, **DFGARCH(1,1)** was the most competitive model for all datasets. Furthermore, DFGARCH(1,1) recover quickly to a crisis, such as the COVID-19 pandemic.

References

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[3] Embrechts P, Kaufmann R, Patie P (2005) Strategic long-term financial risks: single risk factors. Computational Optimization and Appications 32(1):61–90

[4] Koenker, R, and Zhao, Q (1996) Conditional quantile estimation and inference for ARCH models. Econometric Theory 12:793-813